B.Sc.
Mathematics
IV Semester

24MT-4:REAL ANALYSIS-II, VECTOR CALCULUS, AND PARTIAL DIFFERENTIAL EQUATION	
Total Contact Hours: 4 hours/week (L: T:P- 4:0:0)	**Credits: 4**
Total Teaching Hours : 60 Hours **Duration of Examination : 3 Hours**	**Max. Marks: 100** **(Internal Assessment Marks: 20** **Semester End Exam Marks: 80)**

Course Learning Objectives:

- Understanding Riemann Integrals.
- Calculating Riemann Integrals.
- Analyzing Functions Using Riemann Integrals.
- Applying Riemann Integrals to Real-World Problems.
- Understanding PDEs, Linear PDEs, Non-Linear PDEs.
- Applications of PDEs, Communication and Problem-Solving Skills.

Course Outcome(CO's):

On successful completion of the course, the student will able to:

- Define and explain the concept of Riemann integrals, including the definition of the Riemann sum and the limit process.
- Calculate Riemann integrals using various techniques, such as the definition of the Riemann integral, the Fundamental Theorem of Calculus, and substitution methods.
- Apply Riemann integrals to solve problems involving area, volume, and other physical quantities.
- Define and explain the concept of PDEs, including the classification of PDEs (linear, nonlinear, homogeneous, inhomogeneous).
- Solve linear PDEs using analytical and numerical methods, including the heat equation, wave equation, and Laplace equation.
- Analyse nonlinear PDEs, including the classification of nonlinear PDEs (Quasilinear, Semi-linear, fully nonlinear).
- Get introduced to the fundamentals of vector differential and integral calculus.
- Get familiar with the various differential operators and their properties.

- Learn the applications of vector calculus.

Units	Description	Hours
UNIT-I	**RIEMANN INTEGRALS:** Real valued functions of single variable, Integral primitive of function, Inequalities, Absolute values, Lower and Upper bounds, Bounded function, Lower and Upper sums, Theorems on Lower and Upper sums, Examples on Lower and Upper sums, Lower and Upper Riemann Integrals, Necessary and Sufficient conditions for Riemann Integrability, Riemann Integral Darboux's theorem, Riemann Integrability of continuous function, Riemann Integrability of monotonic function.	**15 hours**
UNIT-II	**PROPERTIES OF REIMAN INTEGRALS:** Properties of Riemann Integrability: Sum, Difference, product, division and modules of an integrable function Fundamental theorem of integral calculus, continuity, and derivability of integrable function, First and Second mean value theorem of integral calculus. Leibnitz's result to evaluate the examples of differentiation under the integral sign.	**15 hours**
UNIT-III	**VECTOR CALCULUS**: Vector Algebra: Multiple products, Scalar triple product, vector triple product, geometrical interpretation, related problems; vector function of a scalar variable, interpretation as a space curve, derivative, tangent, normal and binormal vectors to a space curve; Curvature and Torsion of a space curve – definitions, derivation, and problems, Serret – Frenet formulae. **Scalar field:** Gradient of a scalar field, geometrical meaning, directional derivative, unit normal using surfaces - tangent plane and normal to the surface; **Vector field**: Divergence and curl of a vector field, geometrical meaning, solenoidal and irrotational fields; Laplacian of a scalar field; Vector identities.	**15 hours**
UNIT-IV	**PARTIAL DIFFERENTIAL EQUATION:** Formation of partial differential equations, Lagrange's equation $Pp + Qq = R$, First order non-linear partial differential equations and finding their complete integral by Charpit's method (without proof) and reducing to standard forms $f(p,q) = 0$, Clairaut's form, $f(p,q,z) = 0, f(x,p) = g(y,q)$. Homogeneous Linear PDE with constant coefficients and their solutions.	**15 hours**

References/Text books:

1. Introduction to Real Analysis by Robert G. Bartle and Donald R. Sherbert (John Wiley & Sons)
2. Real Analysis: A First Course by Russell A. Gordon (Pearson Education)

3. Elements of Real Analysis, Shanti Narayana, Dr. M. D. Raisinghania, S. Chand
4. G K Ranganath, Text Book of B.Sc. Mathematics, S Chand & Company.
5. Ordinary and Partial Differential Equations by Dr. M. D. Raisinghania, (S. Chand).
6. Partial Differential Equations: An Introduction by Walter A. Strauss (John Wiley & Sons)
7. Partial Differential Equations: A First Course by Robert C. McOwen (Pearson Education)
8. Introduction to Partial Differential Equations by David Borthwick (Springer)
9. Partial Differential Equations: A Modern Approach by Jonathan David Logan (Springer)
10. PDEs: A Practical Approach by Mark S. Gockenbach (SIAM)
11. M. D. Raisinghania, Vector Calculus, S Chand Co. Pvt. Ltd., 2013.
12. M. Spiegel, Vector Analysis, 2 nd Edition, Schaum's Outline Series, Mc-Graw Hill, Education, 2017.
13. C. E. Weatherburn, Elementary Vector Analysis, Alpha edition, 2019.

PRACTICAL PAPER 24MP- 4 - MATHEMATICS LAB-IV	
Total Contact Hours: 4 hours/week (L: T:P- 0:0:4)	**Credits: 2**
Total Teaching Hours : 60 Hours **Duration of Examination : 3 Hours**	**Max. Marks: 50** **(Internal Assessment Marks: 10** **Semester End Exam Marks: 40)**

Course Learning Objectives:

- Develops foundational skills in programming concepts and methodologies.
- Enhances understanding of mathematical concepts through interactive software-based experiments.
- Strengthens programming fundamentals and logical thinking.
- Develops understanding of how algebra and calculus are used in practical scenarios through hands-on experiences with FOSS.

Course Outcome (CO's):

On successful completion of the course, the student will able to:

- Acquire knowledge of FOSS tools and their applications in computer programming.
- Develop and improve coding skills using Programming Language.
- Develop skills in applying software tools to mathematical concepts.
- Build a strong foundation in programming principles and practices.
- Understand the impact of mathematics on science, technology, engineering, and mathematics (STEM) fields.
- Build a strong foundation for future learning and exploration across multiple disciplines.

Syllabus: Problems from 24MT-4 (Theory) may be solved with the help of programming.

Suggested Software's: Maxima/Scilab/Python.

List of Programs (Suggested):

1. Evaluation of Upper and Lower Riemann Sums.
2. Evaluation of Upper and Lower Riemann Integrals.
3. Verification of Euler's theorem for homogeneous functions

4. Verification of Euler's Extension theorem for homogeneous functions
5. Construction of series using Maclaurin's expansion for functions of two variables.
6. Solutions of PDE of the form $Pp + Qq = R$
7. Solution of the first order non-linear PDEs of TYPE-I
8. Solution of the first order non-linear PDEs of TYPE-II
9. Solution of the first order non-linear PDEs of TYPE-III
10. Solution of the first order non-linear PDEs of TYPE-IV
11. Solutions of non-linear PDE by Charpit's Method
12. Solution of homogeneous linear PDEs with constant coefficients
13. Program to find the gradient and Laplacian of a scalar function, divergence and curl of a vector function
14. Program to demonstrate the physical interpretation of gradient, divergence and curl.
15. Program to evaluate a vector line integral.
16. Program to evaluate surface/ volume integral.
17. Program to verify Green's theorem.