

	Subject / Paper Code	Title of the Paper	Instruction Hrs/ Week	Marks			Credits	Examination duration (Hrs)
				Examination	Internal Assessment	Total Marks		
SEMESTER - IV	HARD CORE PAPERS							
	HCT	Functional Analysis	4	80	20	100	4	3
	HCT 4.2	Complex Analysis - II	4	80	20	100	4	3
	HCT 4.3	Differential Geometry	4	80	20	100	4	3
	HCT 4.4	Numerical Methods-II	4	80	20	100	4	3
	SOFT CORE PAPER (ANY ONE)							
	SCT 4.1	Magnetohydrodynamics	4	80	20	100	4	3
		Mathematical Methods						
	PROJECT WORK/REPORT WRITING							
	HCP 4.1	Project	4	80	20	100	4	3

Semester-IV	Subject / Paper Code	Title of the Paper	Instruction Hrs/ Week	Marks			Credits	Examination duration (Hrs)
				Examination	Internal Assessment	Total Marks		
	HCT4.1	Functional Analysis	4	80	20	100	4	3

**Course Objective(s):**

- Students will learn the basic concepts and theorems of functional analysis and its applications.
- The student is able to apply knowledge of functional analysis to solve mathematical problems.
- The student is able to apply knowledge of theorems to solve basic problems.
- To gain understanding of the functional analysis and definition and main properties.

**Course Outcome(s):**

Upon the successful completion of the course, students will be able to

- describe properties of normed linear spaces and construct examples of such spaces.
- understand the Hahn-Banach theorems, the Open Mapping Theorem and its applications.
- apply basic theoretical techniques to analyze linear functionals and operators on Banach and Hilbert spaces.
- obtain Orthogonal complements, Orthonormal sets and conjugate space.

**Syllabus**

**Unit I:** Norm on a linear space over  $F$  (either  $\mathbb{R}$  or  $\mathbb{C}$ ), Banach space. Examples. Norm on quotient space. Continuous linear transformation of normed linear space. The Banach space  $B(N, N')$  for Banach spaces,  $N, N'$ .

**Unit II:** Dual space of a normed linear space. Equivalence of norms. Dual space of  $C[a, b]$ . Isometric isomorphisms. Hahn – Banach theorem and its applications. Separable normed linear spaces.

**Unit III:** Canonical embedding of  $N$  into  $N^{**}$ . Reflexive spaces, Open mapping theorem, closed graph theorem, principle of uniform boundedness (Banach – Steinhaus theorem) projection on Banach spaces.

**Unit- IV:** Hilbert spaces: definition and examples. Orthogonal complements. Orthonormal basis, Gram – Schmidt process of orthonormalization. Bessel's inequality, Riesz – Fisher theorem. Adjoint of an operator. Self – adjoint, normal, unitary and projection operators.

**REFERENCES:**

1. G. F. Simmons: *Introduction to Topology and Modern Analysis*, McGraw Hill Book Com. Inc., 1963.
2. C. Goffman and G. Pedrick: *First Course in Functional Analysis*, Prentice Hall of India Pvt. Ltd. New Delhi (1974)
3. B. V. Limaye: *Functional Analysis*, 2<sup>nd</sup> Edition, New Age International (P) Ltd. Publications (1997)
4. D. Somasundaram: *Functional Analysis*, S. Vishwanathan (Printers & Publishers) Pvt. Ltd. (1994)

Semester-IV	Subject / Paper Code	Title of the Paper	Instruction Hrs/ Week	Marks			Credits	Examination duration (Hrs)
				Examination	Internal Assessment	Total Marks		
	<b>HCT 4.2</b>	<b>Complex Analysis -</b>	<b>4</b>	<b>80</b>	<b>20</b>	<b>100</b>	<b>4</b>	<b>3</b>

### Course Objective(s):

- Students will learn Maximum and minimum modulus principle.
- Students will learn Open mapping theorem and some related theorems.
- Students will learn Conformal mapping.
- Students will learn Analytic continuation.

### Course Outcome(s):

Upon the successful completion of the course, students will be able to

- write proofs of maximum and minimum modulus principle.
- distinguish and utility of open mapping theorem.
- know a conformal mapping and cross ratios.
- apply Riemann mapping theorem.

### Syllabus

**Unit I:** Maximum Modulus Principle. Minimum Modulus Principle. Schwarz's Lemma. Some applications of Schwarz's Lemma. Basic properties of univalent functions.

**Unit II:** Open Mapping Theorem. Deduction of Maximum Modulus Principle using Open Mapping theorem. Hadamard's Three Circle theorem.

**Unit III:** Conformal Mapping. Linear transformations. Unit disc transformations. Sequences and series of functions. Normal families.

**Unit IV:** Weierstrass theorem, Hurwitz's theorem. Montel's theorem. Riemann mapping theorem. Analytic continuation of functions with natural boundaries. Schwarz's reflection principle.

## REFERENCES

1. L. V. Ahlfors: *Complex Analysis*, 3rd ed. New York, McGraw-Hill, 1979.
2. J. B. Conway: *Functions of One Complex Variable*, 2nd edition, Graduate Texts in Mathematics, Springer-Verlag, New York–Berlin, 1978; first edition, 1973.
3. S. Ponnusamy: *Foundations of Complex Analysis*, 2<sup>nd</sup> Edition, Narosa Publishing House, India, 2005.
4. R. V. Churchill and J. W. Brown: *Complex Variables and Applications*, 4<sup>th</sup> Edition, McGraw Hill Book Company, New York, 1984.
5. H. S. Kasana: *Complex Variables- Theory and Applications*, 2<sup>nd</sup> edition, PHI Learning Pvt. Ltd., India, 2005.
6. H. A. Priestley: *Introduction to Complex Analysis*, 2<sup>nd</sup> Edition, Oxford University Press, Indian Edition, 2003.
7. S. L. Segal: *Nine Introductions in Complex Analysis*, revised edition, North-Holland Mathematics Studies, Elsevier, Amsterdam, 2008; first edition, 1981.
8. I. Stewart and D. Tall: *Complex Analysis*, Cambridge University Press, 1983.

Semester-IV	Subject / Paper Code	Title of the Paper	Instruction Hrs/ Week	Marks			Credits	Examination duration (Hrs)
				Examination	Internal Assessment	Total Marks		
	<b>HCT 4.3</b>	<b>Differential</b>	<b>4</b>	<b>80</b>	<b>20</b>	<b>100</b>	<b>4</b>	<b>3</b>

### Course Objective(s):

- The course introduces the fundamentals of differential geometry primarily by focusing on the theory of curves and surfaces in three space.
- To familiarize the students with basic concepts of differential geometry as the subject has got application in general theory of relativity, cosmology and other related disciplines.
- To develop the problem-solving skills arising in geometry by using the techniques of differential calculus and integral calculus.
- To solve real life problems by thinking logically about curves and surfaces.

### Course Outcome(s):

Upon the successful completion of the course, students will be able to

- to give the basic knowledge of classical differential geometry of curves and surfaces in.
- to develop arguments in the geometric description of curves and surfaces in.
- get introduced to the notion of Serret-Frenet frame for space curves and the involutes and evolutes of space curves with the help of examples.
- get knowledge towards the notion of Frenet-Serret Formulae (i.e., compute the curvature and torsion of space curves) with the help of examples.

### Syllabus

**Unit I:** Introduction, Euclidean space, Tangent vectors, Vector fields, Directional derivatives, curves in  $E^3$ , 1 – Forms, differential forms, Mappings on Euclidean

spaces, derivative map, dot product in  $E^3$ , dot product of tangent vectors, Frame at a point.

**Unit II:** Cross product of tangent vectors, curves in  $E^3$ , arc length, reparameterization, The Frenet formulas, Frenet frame field, curvature and torsion of a unit speed curve. Arbitrary speed curves, Frenet formulas for arbitrary speed curve, Covariant derivatives, Frame field on  $E^3$ , connection forms of a frame field, Cartan's structural equations.

**Unit III:** Isometry in  $E^3$ , Derivative map of isometry in  $E^3$ , Calculus on a surface, co-ordinate patch, proper patch, surface in  $E^3$ , Monge patch, Patch computations, parametrization of a cylinder, Differentiable functions and tangent vectors, tangent to a surface, tangent plane, Vector-field, tangent and normal vector-fields on a surface.

**Unit-IV:** Mapping of surfaces, topological properties of surfaces, manifolds. Shape operators, normal curvature, Gaussian curvature, computational techniques, special curves in surfaces.

#### REFERENCES:

1. Barrett. O. Neill, *Elementary Differential Geometry*, Academic Press, New York (1998)
2. T.J. Willmore, *An introduction to Differential Geometry*, Oxford University Press (1999)
3. N.J. Hicks, *Notes on Differential Geometry*, Van Nostrand, Princeton (2000)
4. Nirmala Prakash, *Differential Geometry - An integrated approach*, Tata McGraw Hill Pub. Co. New Delhi (2001).
5. M. P. Do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976.
6. J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer (Undergraduate Texts in Mathematics), 1979.
7. L. P. Eisenhart, *A Treatise on the Differential Geometry of Curves and Surfaces*, Ginn and Company, Boston, 1909.
8. A. Gray, *Differential Geometry of Curves and Surfaces*, CRC Press, 1998.

Semester-IV	Subject / Paper Code	Title of the Paper	Instruction Hrs/ Week	Marks			Credits	Examination duration (Hrs)
				Examination	Internal Assessment	Total Marks		
	<b>HCT 4.4</b>	<b>Numerical Methods -</b>	<b>4</b>	<b>70</b>	<b>30</b>	<b>100</b>	<b>4</b>	<b>3</b>

### Course Objective(s):

- This introductory course presents students some classical and commonly used numerical methods in various disciplines involving computing and numerical approximation and solution of equations.
- To teach theory and applications of numerical methods in linear systems, finding eigenvalues, eigenvectors, interpolation and applications, solving ODEs, PDEs and dealing with statistical problems like testing of hypotheses.
- To lay foundation of computational mathematics for specialized studies and research
- To develop the mathematical skills of the students in the areas of numerical methods.

### Course Outcome(s):

Upon the successful completion of the course, students will be able to



- Solve boundary value problems method of undetermined coefficients, finite difference methods, shooting method, and midpoint method.
- Work out numerical differentiation and integration whenever and wherever routine methods are not applicable.
- Work out on boundary value problems method of undetermined coefficients, finite difference methods, shooting method, and midpoint method.
- Work numerically on the partial differential equations using different methods through the theory of finite differences.

## Syllabus

**UNIT-I: Numerical Differentiation and Integration:** Introduction, errors in numerical differentiation, extrapolation methods, cubic spline method, differentiation formulae with function values, maximum and minimum values of a tabulated function, partial differentiation. Numerical Integration, Newton-Cotes integration methods; Trapezoidal rule, Simpson's  $1/3^{\text{rd}}$  rule, Simpson's  $3/8^{\text{th}}$  rule and Weddle's rule. Gaussian integration methods and their error analysis. Gauss-Legendre, Gauss-Hermite, Gauss-Laguerre and Gauss-Chebyshev integration methods and their error analysis. Romberg integration, Double integration.

**UNIT-II: Numerical Solutions of Initial Value Problems (Ordinary Differential Equations):** Introduction, Derivation of Taylor's series method, Euler's method, Modified Euler Method, Runge-Kutta Second, Third and Forth order methods, Runge-Kutta-Gill method, Predictor-Corrector methods; Milne's method, Adam's Bashforth Moulton method.

**UNIT-III: Solutions of Boundary Value Problems (Ordinary Differential Equations):** Introduction, solution of boundary value problems method of undetermined coefficients, finite difference methods, shooting method, and midpoint method.

**UNIT - IV: Numerical Solutions of Partial Differential Equations:** Introduction, derivation of finite difference approximations to the derivatives, solution of Laplace equation by Jacobi, Gauss Seidel and SOR methods, ADI method, Parabolic, solution of heat equation by Schmidt and Crank-Nicolson methods, solution of wave equation using finite difference method.

## REFERENCES:

1. S. Larsson and V. Thomee: *Partial differential equations with numerical methods*, Springer, 2008.
2. J. W. Thoma: *Numerical partial differential equations: finite difference methods*, 2<sup>nd</sup> edition, pringer, 1998.
3. R. K. Jain, S. R. K. Iyengar and M. K. Jain: *Numerical methods for scientific and Engineering computation*, Wiley Eastern, 2001.
4. S. D. Conte and Carl De Boor: *Elementary Numerical Analysis*, McGraw Hill, 2000.
5. M. K. Jain: *Numerical Solution of Differential Equations*, Wiley Eastern, 1990.
6. G. D. Smith: *Numerical Solution of PDE*, Oxford University Press, 1998.

7. A. Iserles: *A first course in the numerical analysis of differential equations*, 2<sup>nd</sup> edition, Cambridge texts in applied mathematics, 2008.
8. R.L. Burden and J.D. Faires: *Numerical Analysis*, 7<sup>th</sup> edition, Thomson-Brooks/Cole, 1989.

Semester-IV	Subject / Paper Code	Title of the Paper	Instruction Hrs/ Week	Marks			Credits	Examination duration (Hrs)
				Examination	Internal Assessment	Total Marks		
	<b>SCT4.1</b>	Magnetohydrodynamics	<b>4</b>	<b>80</b>	<b>20</b>	<b>100</b>	<b>4</b>	<b>3</b>

**Course Objective(s):**

- Students will learn the basic concepts of boundary layer theory and its applications
- Students will learn the fundamentals of Magnetohydrodynamics, which include theory of Maxwell's equations, basic equations, exact solutions and applications of classical MHD.
- Give students practice in concepts of dimensional analysis and problem solving.
- Students will learn the applications of Magnetohydrodynamics in daily life.

**Course Outcome(s):**

Upon the successful completion of the course, students will be able to

- understand the concept of boundary layer theory and its applications
- provide the details of the derivation of ideal and resistive MHD equations.
- demonstrate the basic properties of ideal MHD.
- provides a theoretical and practical background to Ph.D. thesis in heat transport and stellar atmosphere models.

**Syllabus**

**UNIT – I: Theory of laminar boundary layer concepts :** Definition of laminar and turbulent, Two dimensional boundary layer equations for flow over a plane wall, Prandtl's boundary layer concept, some definition of boundary layer thickness, displacement thickness, momentum thickness. Boundary layer flow along a flat plate- Blasius solution.

**UNIT – II: Basic equations of MHD:** Outline of basic equations of MHD, (i) Conservation of mass (ii) Conservation of momentum .Lagrangian approach and Eulerian approach. Magnetic Induction equation, Lorentz force. Exact Solutions: Hartmann flow, isothermal boundary conditions, Temperature distribution in Hartmann flow, Hartmann-Couette flow.

**UNIT – III: Dimensional analysis:** Dimensional homogeneity, Rayleigh's technique, Buckingham  $\pi$ - theorem, model analysis and dynamical similarity, Reynolds's number, significance of Reynold's number. Some useful dimensionless number: (i) Reynolds's number and magnetic Reynolds's number (ii) Froude number (ii) Euler number (iv) Mach number (v) Prandtl number and magnetic Prandtl number (vi) Eckert number.

**UNIT – IV: Convective instability:** Basic concepts of convective instability, Rayleigh Bénard convection, Boussinesq approximation, equation of state,

perturbed state, normal modes, principle of exchange of stabilities, first variation principle, different boundary conditions on velocity and temperature, solution for free-free boundaries.

## **REFERENCES:**

1. Schlichting H., Boundary layer theory, McGraw-Hill, 1979.
2. Lin C. C., The theory of Hydrodynamic stability, Cambridge University Press, 1955.
3. Chandrasekhar S., Hydrodynamic and Hydrodynamic stability, Oxford University Press. 1961.
4. G. K. Batchelor: An Introduction to Fluid Mechanics, Foundation Books, New Delhi, (1994).
5. D. J. Tritton, Physical fluid dynamics, Oxford Science publication, second edition, 1987.
6. Nield D. A. and Bejan A., Convection in porous media, Springer, 2006.
7. F. Chorlton: Text Book of Fluid Dynamics, CBS Publishers, New Delhi, (1985).
8. R. K. Rathy: An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi, (1976).

Semester-IV	Subject / Paper Code	Title of the Paper	Instruction Hrs/ Week	Marks			Credits	Examination duration (Hrs)
				Examination	Internal Assessment	Total Marks		
	SCT4.1	Mathematical Methods	4	80	20	100	4	3

#### Course Objective(s):

- Understand the concepts of Asymptotic expansion of functions, power series as asymptotic series, asymptotic forms for large and small variables.
- Find the solutions for Linear equation with variable coefficients and nonlinear BVP's.
- Problems involving Boundary layers.
- Providing a set of powerful analytical tools for the solution of problems.

#### Course Outcome(s):

Upon the successful completion of the course, students will be able to

- recognize the different methods of finding solutions for integral equations by separable kernel, Neumann's series resolvent kernel and transform methods.
- apply the knowledge of Integral Equations and Integral transforms in finding the solutions of differential equations, initial value problems and boundary value problems.
- perform analysis on Regular and singular perturbation methods.
- perform analysis of first and second order differential equations involving constant and variable coefficients.

## Syllabus

**Unit – I: Integral Transforms:** General definition of integral transforms, Kernels, etc.

Hankel transforms to solve ODEs and PDEs - typical examples. Discrete Orthogonality and Discrete Fourier transform. Wavelets with examples, wavelet transforms.

**Unit – II: Integral Equations:** Definition, Volterra and Fredholm integral equations. Solution by separable kernel, Neumann's series resolvent kernel and transform methods, Convergence for Fredholm and Volterra types. Reduction of IVPs, BVPs and eigen value problems to integral equations. Hilbert Schmidt theorem, Raleigh Ritz and Galerkin methods.

**Unit – III: Asymptotic Expansions:** Asymptotic expansion of functions, power series as asymptotic series, Asymptotic forms for large and small variables. Uniqueness properties and Operations. Asymptotic expansions of integrals; Method of integration by parts (include examples where the method fails), Laplace's method and Watson's lemma, method of stationary phase and steepest descent.

**Unit – IV: Perturbation methods:** Regular and singular perturbation methods: Parameter and co-ordinate perturbations. Regular perturbation solution of first and second order differential equations involving constant and variable coefficients. Include Duffing's equation, Van der Pol oscillator, small Reynolds number flow. Singular perturbation problems, Matched asymptotic expansions, simple examples. Linear equation with variable coefficients and nonlinear BVP's. Problems involving Boundary layers.

## REFERENCES:

1. IN Sneddon: *The use of Integral Transforms*, Tata Mc Graw Hill, Publishing Company Ltd, New Delhi, 1974.
2. R.P. Kanwal: *Linear integral equations theory & techniques*, Academic Press, New York, 1971.
3. C.M. Bender and S.A. Orszag: *Advanced mathematical methods for scientists and engineers*, Mc Graw Hill, New York, 1978.
4. H.T. Davis: *Introduction to nonlinear differential and integral equations*, Dover Publications, 1962.
5. A.H. Nayfeh: *Perturbation Methods*, John Wiley & Sons, New York, 1973.
6. D. Hong, J. Wang and R. Gardner: *Real analysis with introduction to wavelets and applications*, Academic Press Elsevier (2006)
7. R.V. Churchill: *Operational Mathematics*, Mc. Graw Hill, New York, 1958.

